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NOTE ON GENERALIZED JORDAN LEFT CENTRALIZERS

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Abstract

Let δ be an additive map from a ring R into an R-bimodule M and $\beta : R \times R \to M$ be a biadditive map satisfying $\beta(x, yz) - \beta(x, y)z - \beta(xy, z) = 0$. We call (δ, β) is a generalized left centralizer (respectively, a generalized Jordan left centralizer), if $\delta(xy) = \delta(x)y + \beta(x, y)$ for all $x, y \in R$ (respectively, $\delta(x^2) = \delta(x)x + \beta(x, x)$ for all $x \in R$). In this paper, we show that every generalized Jordan left centralizer is a generalized left centralizer under certain conditions. In Corollary 3.4, we apply generalized Jordan left centralizers to generalized Jordan derivations on certain rings and operator algebras.

1. Introduction

Let δ be an additive map from a ring R into an R-bimodule M and x, y be arbitrary elements of R. δ is called a *left centralizer* (respectively, *Jordan left centralizer*), if $\delta(xy) = \delta(x)y$ (respectively, $\delta(x^2) = \delta(x)x$). δ is called a *generalized derivation* (respectively,

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generalized Jordan derivation), if there exists a derivation (respectively, Jordan derivation) $\tau : R \to M$ such that $\delta(xy) = \delta(x)y + x\tau(y)$ (respectively, $\delta(x^2) = \delta(x)x + x\tau(x)$). We denote it by (δ, τ). Generalized derivations and generalized Jordan derivations were introduced by Bresar [3] and their properties have been discussed in many papers.

Nakajima [9] introduced a new type of generalized derivations and generalized Jordan derivations associate with Hochschild 2-cocycles. A biadditive map $\alpha : R \times R \to M$ is called a *Hochschild 2-cocycle*, if

$$x\alpha(y, z) - \alpha(xy, z) + \alpha(x, yz) - \alpha(x, y)z = 0. \tag{1.1}$$

An additive map $\delta : R \to M$ is called a *generalized derivation associate with Hochschild 2-cocycle,* if there exists a 2-cocycle α such that

$$\delta(xy) = \delta(x)y + x\delta(y) + \alpha(x, y), \tag{1.2}$$

and δ is called a *generalized Jordan derivation associate with Hochschild 2-cocycle*, if

$$\delta(x^2) = \delta(x)x + x\delta(x) + \alpha(x, x). \tag{1.3}$$

We denote it by (δ, α).

Motivated by Nakajima's definitions, we will introduce a type of generalized left centralizers and generalized Jordan left centralizers. An additive map $\delta : R \to M$ is called a *generalized left centralizer*, if there exists a biadditive map $\beta : R \times R \to M$ satisfying

$$\beta(x, yz) - \beta(x, y)z - \beta(xy, z) = 0, \tag{1.4}$$

such that

$$\delta(xy) = \delta(x)y + \beta(x, y), \tag{1.5}$$

and δ is called a *generalized Jordan left centralizer*, if there exists a biadditive map $\beta : R \times R \to M$ satisfying (1.4) such that

$$\delta(x^2) = \delta(x)x + \beta(x, x). \tag{1.6}$$

We denote it by (δ, β).

Remark. It is easy to show that (δ, β) is a generalized left centralizer, if and only if (δ, α) is a generalized derivation associate with Hochschild 2-cocycle, where $\alpha(x, y) = \beta(x, y) - x\delta(y)$. One of the problem is whether generalized Jordan left centralizers and generalized Jordan derivations associate with Hochschild 2-cocycles are equivalent. In Corollary 3.2, we obtain some results.

In recent years, there have been a number of papers on the study of derivations, left centralizers, and generalized derivations. Herstein [5, Theorem 3.1] first proved that a Jordan derivation of 2-torsion free prime rings is a derivation and Bresar [2, Theorem 1] extended this result into 2-torsion free semiprime rings. Zalar [11, Proposition 1.4] proved that a Jordan left centralizer of 2-torsion free semiprime rings is a left centralizer. In [1, 6, 10], authors showed that every generalized Jordan derivation on 2-torsion free rings, which have a commutator nonzero divisor, 2-torsion free semiprime rings and nest algebras is a generalized derivation. In [9, Theorem 6], Nakajima showed that every generalized Jordan derivation associate with Hochschild 2-cocycle is a generalized derivation under certain conditions.

In this paper, we show that every generalized Jordan left centralizer is a generalized left centralizer under certain conditions. In Corollary 3.4, we easily show that a generalized Jordan derivation on 2-torsion free semiprime rings, 2-torsion free rings, which have a commutator nonzero divisor, CSL algebras, triangular algebras or a class of reflexive algebras $\operatorname{alg}\mathcal{L}$ on a Banach space X satisfying $\vee\{L \in \mathcal{L} : L_- < X\} = X$ or $\wedge \{L_- : L \in \mathcal{L}, L > (0)\} = (0)$ is a generalized derivation.

2. Lemmas

In the following, we assume that R is a ring with the center $\mathcal{Z}$ and M is an R-bimodule, unless otherwise stated. To obtain our results, we need the following lemmas:

Lemma 2.1. *Let R be a ring and M be a 2-torsion free R-bimodule. If $(\delta, \beta) : R \to M$ is a generalized Jordan left centralizer, then the following hold*:

(1) $\delta(xy + yx) = \delta(x)y + \delta(y)x + \beta(x, y) + \beta(y, x)$,

(2) $\delta(xyx) = \delta(x)yx + \beta(x, yx)$,

(3) $\delta(xyz + zyx) = \delta(x)yz + \delta(z)yx + \beta(x, yz) + \beta(z, yx)$,

for all $x, y, z \in R$.

Proof. (1) Replacing x by $x + y$ in (1.6), (1) is easily obtained.

(2) Replacing y by $xy + yx$ in (1), we have

$$\begin{aligned}
&\delta(x^2y + xyx + xyx + yx^2) \\
&\quad = \delta(x)(xy + yx) + \delta(xy + yx)x + \beta(x, xy + yx) + \beta(xy + yx, x) \\
&\quad = \delta(x)xy + \delta(x)yx + \delta(x)yx + \delta(y)x^2 + \beta(x, y)x + \beta(y, x)x \\
&\qquad + \beta(x, xy) + \beta(x, yx) + \beta(xy, x) + \beta(yx, x).
\end{aligned}$$

On the other hand,

$$\begin{aligned}
&\delta(x^2y + xyx + xyx + yx^2) \\
&\quad = \delta(x)xy + \beta(x, x)y + \delta(y)x^2 + \beta(x^2, y) + \beta(y, x^2) + 2\delta(xyx).
\end{aligned}$$

Hence,

$$\begin{aligned}
2\delta(xyx) = {} & \{\beta(x, xy) - \beta(x, x)y - \beta(x^2, y)\} + \{\beta(y, x)x + \beta(yx, x) - \beta(y, x^2)\} \\
& + \{\beta(x, y)x + \beta(x, yx) + \beta(xy, x)\} + 2\delta(x)yx.
\end{aligned}$$

Since β satisfies (1.4), we have

$$\beta(x, xy) - \beta(x, x)y - \beta(x^2, y) = 0,$$

$$\beta(y, x)x + \beta(yx, x) - \beta(y, x^2) = 0,$$

and

$$\beta(x, yx) = \beta(xy, x) + \beta(x, y)x.$$

As M is 2-torsion free, we obtain $\delta(xyx) = \delta(x)yx + \beta(x, yx)$.

(3) Replacing x by $x + z$ in (2) gives (3). □

Now for $x, y \in R$, we set

$$F(x, y) = \delta(xy) - \delta(x)y \quad \text{and} \quad B(x, y) = F(x, y) - \beta(x, y).$$

Then $F(x, y)$ and $B(x, y)$ are biadditive and by Lemma 2.1 (1), we have

$$B(x, y) + B(y, x) = 0. \tag{2.1}$$

Lemma 2.2. *Let R be a ring and M be a 2-torsion free R-bimodule. If $(\delta, \beta) : R \to M$ is a generalized Jordan left centralizer, then the following hold*:

(1) $B(x, y)z[x, y] = 0$,

(2) $B(x, y)[x, y] = 0$,

for all $x, y, z \in R$.

Proof. (1) By Lemma 2.1 (3),

$$\begin{aligned}\delta((xy)z(yx) + (yx)z(xy)) &= \delta(xy)zyx + \delta(yx)zxy + \beta(xy, zyx) + \beta(yx, zxy)\\ &= F(x, y)zyx + \delta(x)yzyx + F(y, x)zxy + \delta(y)xzxy\\ &\quad + \beta(xy, zyx) + \beta(yx, zxy).\end{aligned}$$

On the other hand, by Lemma 2.1 (2),

$$\delta(x(yzy)x + y(xzx)y) = \delta(x)yzyx + \beta(x, yzyx) + \delta(y)xzxy + \beta(y, xzxy).$$

Thus,

$$F(x, y)zyx + F(y, x)zxy + \beta(xy, zyx) + \beta(yx, zxy) = \beta(x, yzyx) + \beta(y, xzxy). \tag{2.2}$$

Since β satisfies (1.4), we have

$$\beta(x, yzyx) = \beta(x, y)zyx + \beta(xy, zyx) \text{ and } \beta(y, xzxy) = \beta(y, x)zxy + \beta(yx, zxy).$$

Substituting the above relations in (2.2), we obtain $B(x, y)zyx + B(y, x)zxy = 0,$ which according to (2.1) implies

$$B(x, y)z[x, y] = 0.$$

(2) By Lemma 2.1 (2) and (3),

$$\begin{aligned} 0 &= \delta((xy)^2 + xy^2x) - \delta(xy(xy) + (xy)yx) \\ &= \delta(xy)xy + \beta(xy, xy) + \delta(x)y^2x + \beta(x, y^2x) \\ &\quad - \delta(x)yxy - \delta(xy)yx - \beta(x, yxy) - \beta(xy, yx) \\ &= F(x, y)[x, y] + \{\beta(xy, xy) - \beta(x, yxy)\} \\ &\quad + \{\beta(x, y^2x) - \beta(xy, yx)\}. \end{aligned} \tag{2.3}$$

Since β satisfies (1.4), we have

$$\beta(xy, xy) - \beta(x, yxy) = -\beta(x, y)xy \text{ and } \beta(x, y^2x) - \beta(xy, yx) = \beta(x, y)yx.$$

Substituting the above relations in (2.3), we obtain

$$0 = F(x, y)[x, y] - \beta(x, y)[x, y] = B(x, y)[x, y]. \qquad \square$$

The following lemma can been found in [9, Lemma 5].

Lemma 2.3. *Let R be a 2-torsion free ring and G_1, G_2 be additive groups. Let $S, T : G_1 \times G_2 \to R$ be biadditive maps. Assume that $S(x_1, x_2)T(x_1, x_2) = 0$ for all $x_i \in G_i, i = 1, 2.$ If there exists a nonzero divisor $T(a_1, a_2)$ for some $a_i \in G_i, i = 1, 2,$ then $S(x_1, x_2) = 0$ for all $x_i \in G_i, i = 1, 2.$*

The following lemmas are useful in dealing with 2-torsion free semiprime rings, which can been found in [11, Lemmas 1.1-1.3].

Lemma 2.4. *Let R be a semiprime ring. If $a, b \in R$ are such that $axb = 0$ for all $x \in R,$ then $ab = ba = 0.$*

Lemma 2.5. *Let R be a semiprime ring and $A, B : R \times R \to R$ be biadditive mappings. If $A(x, y)\omega B(x, y) = 0$ for all $x, y, \omega \in R$, then $A(x, y)\omega B(u, v) = 0$ for all $x, y, \omega, u, v \in R$.*

Lemma 2.6. *Let R be a semiprime ring with the center $\mathcal{Z}$ and $a \in R$ be some fixed element. If $a[x, y] = 0$ for all $x, y \in R$, then there exists an ideal $\mathcal{U}$ of R such that $a \in \mathcal{U} \subset \mathcal{Z}$ holds.*

3. Generalized Jordan Left Centralizers

The following theorem is our main result:

Theorem 3.1. *Let R be a ring and $(\delta, \beta) : R \to R$ be a generalized Jordan left centralizer. If R satisfies one of the following conditions, then (δ, β) is a generalized left centralizer:*

(1) *R has an identity* 1.

(2) *R is a* 2*-torsion free ring and there exist $a, b \in R$ such that $[a, b]$ is a nonzero divisor.*

(3) *R is a* 2*-torsion free semiprime ring.*

Proof. (1) By Lemma 2.1 (1), we obtain

$$\delta(xy + yx) = \delta(x)y + \delta(y)x + \beta(x, y) + \beta(y, x).$$

Taking $y = 1$ in the above equation, we have

$$\delta(x) = \delta(1)x + \beta(x, 1) + \beta(1, x).$$

Since $\beta(x, 1) = 0$ by (1.4), we have

$$\delta(x) = \delta(1)x + \beta(1, x),$$

for all $x \in R$. Hence, for any $x, y \in R$,

$$\delta(xy) = \delta(1)xy + \beta(1, xy).$$

By (1.4), $\beta(1, xy) = \beta(1, x)y + \beta(x, y)$. So

$$\delta(xy) = \delta(x)y + \beta(x, y).$$

(2) By Lemma 2.2 (2), $B(x, y)[x, y] = 0$ for all $x, y \in R$. Since there exist $a, b \in R$ such that $[a, b]$ is a nonzero divisor and $B(x, y)$ and $[x, y]$ are biadditive maps, then by Lemma 2.3, $B(x, y) = 0$.

(3) By Lemmas 2.2 (1) and 2.5, we have $B(x, y)z[u, v] = 0$ for all $x, y, z, u, v \in R$, then by Lemma 2.4, $B(x, y)[u, v] = 0$ for all $x, y, u, v \in R$. Now, fix some $x, y \in R$ and write B instead of $B(x, y)$ to simplify further writing. Our goal is to prove $B = 0$. By Lemma 2.6, there exists an ideal $\mathcal{U}$ of R such that $B \in \mathcal{U} \subset \mathcal{Z}$. Since $\mathcal{U}$ is an ideal, $bB, Bb \in \mathcal{Z}$ for all $b \in R$. This gives us

$$xB^2y = xyB^2 = yB^2x \quad \text{and} \quad xyB^2 = xByB.$$

By Lemma 2.1 (1),

$$\begin{aligned}
&2\delta(x(B^2y) + (B^2y)x) \\
&= 2\delta(x)B^2y + 2\delta(B^2y)x + 2\beta(x, B^2y) + 2\beta(B^2y, x) \\
&= 2\delta(x)B^2y + \delta(B^2y + yB^2)x + 2\beta(x, B^2y) + 2\beta(B^2y, x) \\
&= 2\delta(x)B^2y + \delta(B)Byx + \beta(B, B)yx + \delta(y)B^2x + \beta(B^2, y)x \\
&\quad + \beta(y, B^2)x + 2\beta(x, B^2y) + 2\beta(B^2y, x).
\end{aligned} \tag{3.1}$$

Since β satisfies (1.4), we have for any $u, v \in R$,

$$\beta(B, B)v + \beta(B^2, v) = \beta(B, Bv), \tag{3.2}$$

$$\beta(u, B^2v) = \beta(u, vB^2) = \beta(u, v)B^2 + \beta(uv, B^2), \tag{3.3}$$

$$\beta(B^2v, u) = \beta(v, B^2u) - \beta(v, B^2)u = \beta(v, u)B^2 + \beta(vu, B^2) - \beta(v, B^2)u. \tag{3.4}$$

Hence by (3.1)-(3.4), we have

$$2\delta(xB^2y + B^2yx)$$

$$= 2\delta(x)B^2y + \delta(B)Byx + \delta(y)B^2x + \beta(B, By)x + 2\beta(x, y)B^2$$

$$+ 2\beta(xy, B^2) + 2\beta(y, x)B^2 + 2\beta(yx, B^2) - \beta(y, B^2)x. \tag{3.5}$$

Similar to the proof of (3.5), we also have

$$2\delta(yB^2x + B^2xy)$$

$$= 2\delta(y)B^2x + \delta(B)Bxy + \delta(x)B^2y + \beta(B, Bx)y + 2\beta(y, x)B^2$$

$$+ 2\beta(yx, B^2) + 2\beta(x, y)B^2 + 2\beta(xy, B^2) - \beta(x, B^2)y. \tag{3.6}$$

Hence by (3.5) and (3.6), it follows that

$$\delta(y)B^2x = \delta(x)B^2y + \beta(B, By)x - \beta(y, B^2)x - \beta(B, Bx)y + \beta(x, B^2)y. \tag{3.7}$$

On the other hand, by Lemma 2.1 (1),

$$4\delta(xyB^2) = 2\delta(xyB^2 + B^2xy) = 2\delta(xy)B^2 + 2\delta(B^2)xy + 2\beta(xy, B^2) + 2\beta(B^2, xy)$$

$$= 2\delta(xy)B^2 + 2\delta(B)Bxy + 2\beta(B, B)xy + 2\beta(xy, B^2) + 2\beta(B^2, xy),$$

then by (3.2) and (3.3), we have

$$4\delta(xyB^2) = 2\delta(xy)B^2 + 2\delta(B)Bxy + 2\beta(B, Bxy) + 2\beta(x, yB^2) - 2\beta(x, y)B^2. \tag{3.8}$$

By Lemma 2.1 (1) again, we have

$$4\delta(xByB) = 2\delta(xByB + yBxB)$$

$$= 2\delta(xB)yB + 2\delta(yB)xB + 2\beta(xB, yB) + 2\beta(yB, xB)$$

$$= \delta(xB + Bx)By + \delta(yB + By)Bx + 2\beta(Bx, By) + 2\beta(By, Bx)$$

$$= \delta(x)B^2y + \delta(B)Bxy + \beta(x, B)By + \beta(B, x)By$$

$$+ \delta(y)B^2x + \delta(B)Bxy + \beta(y, B)Bx + \beta(B, y)Bx$$

$$+ 2\beta(Bx,\ By) + 2\beta(By,\ Bx). \tag{3.9}$$

Since for any $u,\ v \in R$,

$$\begin{aligned}\beta(u,\ B)v + \beta(B,\ u)v &= \beta(u,\ Bv) - \beta(uB,\ v) + \beta(B,\ uv) - \beta(Bu,\ v) \\ &= \beta(u,\ Bv) + \beta(B,\ uv) - 2\beta(Bu,\ v). \end{aligned} \tag{3.10}$$

So by (3.9) and (3.10), we obtain

$$\begin{aligned}4\delta(xByB) &= \delta(x)B^2y + \delta(y)B^2x + 2\delta(B)Bxy \\ &\quad + \beta(x,\ B^2y) + \beta(y,\ B^2x) + 2\beta(B,\ Bxy). \end{aligned} \tag{3.11}$$

Hence by (3.7), (3.8), and (3.11), we have

$$\begin{aligned}&2\delta(xy)B^2 - 2\beta(x,\ y)B^2 - 2\delta(x)yB^2 \\ &= \beta(y,\ B^2x) + \beta(B,\ By)x - \beta(y,\ B^2)x \\ &\quad - \beta(B,\ Bx)y + \beta(x,\ B^2)y - \beta(x,\ yB^2). \end{aligned} \tag{3.12}$$

Since β satisfies (1.4), it follows that

$$\begin{aligned}\beta(y,\ B^2x) &= \beta(y,\ B^2)x + \beta(yB^2,\ x) \\ &= \beta(y,\ B^2)x + \beta(B,\ Byx) - \beta(B,\ By)x \\ &= \beta(y,\ B^2)x - \beta(B,\ By)x + \beta(B,\ Bx)y + \beta(B^2x,\ y) \\ &= \beta(y,\ B^2)x - \beta(B,\ By)x + \beta(B,\ Bx)y \\ &\quad + \beta(x,\ B^2y) - \beta(x,\ B^2)y. \end{aligned} \tag{3.13}$$

Finally, we arrive at $B^3 = 0$ by (3.12) and (3.13). Hence

$$B^2RB^2 = B^4R = 0,$$

$$BRB = B^2R = 0,$$

which implies $B = 0$ and the proof is complete. $\square$

Corollary 3.2. (1) *If R is as in Theorem* 3.1, *then* (δ, β) *is a generalized Jordan left centralizer implies that* (δ, α) *is a generalized Jordan derivation associate with Hochschild* 2-*cocycle, where* $\alpha(x, y) = \beta(x, y) - x\delta(y)$.

(2) *If R is as in* [9, *Theorem* 6], *then* (δ, α) *is a generalized Jordan derivation associate with Hochschild* 2-*cocycle implies that* (δ, β) *is a generalized Jordan left centralizer, where* $\beta(x, y) = \alpha(x, y) + x\delta(y)$.

In particular, if R is a 2-*torsion free non-commutative prime ring or as in Theorem* 3.1 (2), *then* (δ, β) *is a generalized Jordan left centralizer*, *if and only if* (δ, α) *is a generalized Jordan derivation associate with Hochschild* 2-*cocycle*.

Proof. Suppose that (δ, β) is a generalized Jordan left centralizer. Let $\alpha(x, y) = \beta(x, y) - x\delta(y)$. Then $\delta(x^2) = \delta(x)x + x\delta(x) + \alpha(x, x)$. By Theorem 3.1, we have

$$\delta(yz) = \delta(y)z + \beta(y, z),$$

for all $y, z \in R$. Hence,

$$x\alpha(y, z) - \alpha(xy, z) + \alpha(x, yz) - \alpha(x, y)z = x(\beta(y, z) - \delta(yz) + \delta(y)z) = 0,$$

for all $x, y, z \in R$. The proof is complete. By [9, Theorem 6], the proof of (2) is similar. □

Corollary 3.3. *Let R be as in Theorem* 3.1 *and* $\delta : R \to R$ *be an additive map such that* $\delta(x^2) = \delta(x)x + x\tau(x)$ *for all* $x \in R$, *where* $\tau : R \to R$ *is a derivation. Then* (δ, τ) *is a generalized derivation.*

Proof. Let $\beta(x, y) = x\tau(y)$. Since τ is a derivation, it is easy to show that β satisfies (1.4). Hence (δ, β) is a generalized Jordan left centralizer. By Theorem 3.1, $\delta(xy) = \delta(x)y + x\tau(y)$ for all $x, y \in R$, that is, δ is a generalized derivation. □

By [2, 4, 7, 8, 12], we have that every Jordan derivation on certain rings and operator algebras is a derivation. Hence by Corollary 3.3, we obtain the following corollary:

Corollary 3.4. *If* (δ, τ) *is a generalized Jordan derivation on* 2*-torsion free semiprime rings*, 2*-torsion free rings*, *which have a commutator nonzero divisor*, *CSL algebras*, *triangular algebras or a class of reflexive algebras* $\operatorname{alg}\mathcal{L}$ *on a Banach space* X *satisfying* $\vee\{L \in \mathcal{L} : L_- < X\} = X$ *or* $\wedge\{L_- : L \in \mathcal{L}, L > (0)\} = (0)$, *then* (δ, τ) *is a generalized derivation*.

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